Word Translations

A lot of the GMAT focus is on testing your ability to translate between English and math, as well as your understanding of what the math means in terms of English.

Table of Contents

Algebraic Translations

Translate English into Algebra correctly. Here are the general steps and some notes to keep in mind when carrying them out. Follow the chocolate bar example to help understand this concept. The work for each step comes after the important information you should keep in mind when carrying out that step.

The Chocolate Bar Example

A candy company sells milk chocolates at $5 per pound and dark chocolates at $4 per a pound. If John buys a 7-pound box of chocolate that cost him $31, how many pounds of dark chocolate are in the box?

Step 1: Assign variables

1. Pick letters that make sense, for example M for Apple, and D for Dogs
2. Always include the units so you don't forget what you're counting
3. Sometimes GMAT problems expect the use of subscripts such as the male and female population of two towns. M_1 F_1, and M_2 F_2
4. ***<u>Always</u>*** write down the **ultimate unknown**, the item you're solving for, and add an equal sign and question mark. If you are solving for the variable Q according to the prompt, then write down <u>"Q=?"</u> and circle it! If the question is asking you to solve for Q plus 10, then you write down <u>"Q+10=?"</u> or you will find yourself falling victim to GMAT traps.
5. Try and minimize the number of variables by making use of relationships. For example, if we have 10 bars of candy split between dark chocolate and milk chocolate, you can represent dark chocolate as D, and milk chocolate as M, **or you could represent dark chocolate as D and milk chocolate as 10-D.**

We know that there are 7 pounds of chocolate total in the box, so we conclude that:
M = Pounds of Milk chocolate
(7 – M) = Pounds of Dark chocolate

<u>(7 – M) = ?</u>

Step 2: Write equation(s)

1. Sometimes you will know the equation you need because it is given or it is a standard equation you've studied. Other times you will need to construct it. Start by expressing a relationship between the unknowns and the known values in words. Then translate these words into mathematical symbol.
2. The GMAT will expect you to know several relationships. These relationships are noted at the end of this section. You must master them.

"Total cost of box is equal to cost of milk chocolate plus the cost of dark chocolate"
Total cost = Cost of Milk + Cost of Dark
Total cost = (Unit Price M)(Quantity of M) + (Unit price D)(Quantity of D)
$31 = (5)(M) + (4)(7 - M)$

Step 3: Solve the algebra

1. Always make sure that whatever you are doing makes sense algebraically. Make sure that you don't forget to follow a rule because you're rushing, like dividing an inequality by a variable when you are not 100% sure whether is positive or negative.

$31 = (5)(M) + (4)(7 - M)$
$31=5M+28 - 4M$
$3 = M$

Step 4: Evaluate the solution in the context of the problem

2. Go back to the problem and make sure that you're actually answering the question that is asked. The GMAT knows that many people will not do this step, so they often put a trap answer into the choices of just the

variable. For example, we know that M=3. But this is NOT the answer to the question, but it would certainly be an answer choice. We must plug this into *(7 – M)* to find the answer of how much dark chocolate. This is why we write down and circle our ultimate unknown.

(7 – M) = Dark chocolate
(7 – 3) = 4
There are 4 pounds of dark chocolate in the box

Relationships You Must Master

I. Total Cost = Unit Price x Quantity Purchased
II. Total Sales = Unit Price x Quantity Sold
  ⇒ Total Sales = Revenue (words are interchangeable)
III. Profit = Revenue – Cost
IV. Unit Profit = Selling Price – Unit Cost
  ⇒ Unit Profit = Markup (words are interchangeable)
V. Total Earnings = Wage Rate x Time
  ⇒ Wage Rate and Time must be in same time units ($ Per Hour x Number Hours, etc)

Finally, note that sometimes you will need to express relationships as inequalities rather than equations.

Translating Words Correctly

- "A is half the size of B" — A = (1/2)(B)
- "A is 5 less than B" — A = B – 5
- "A is less than B" — A < B
- "Time bought twice as many apples as bananas" — A = 2B
- P is X percent of Q — P = (X/100)Q or P/Q = X/100

Always write percentages as a variable over 100

Use Charts to Organize Variables

Sometimes you need to organize your variables using charts; the chart is also effective in showing relationship and helping to find an equation. This will happen with many different types of problems including those dealing with work, rates, distances, and so on. A common problem on the GMAT is the age problem. We will demonstrate the chart-variable method with the age problem.

8 years ago Tim was half as old as Kate. Kate is now 20 years older than Tim. How old will Tim be 10 years from now?

We could use two variables, one for Tim and one for Kate. But we could also use a chart to help us organize information and possibly use one variable. Here is how our chart looks.

	Now – 8yrs	**Now**	**Now + 10yrs**
Tim			
Kate			

Ultimate unknown = Tim 10 years from now?

Step 1: Write variables (using chart)

	Now – 8yrs	**Now**	**Now + 10yrs**
Tim	T-8	T	T+10

Kate	(T+20)-8	(T+20)	(T+20)+10

We have decided to let T = Tim's age. Because we knew that Kate currently is 20 years older than Tim, we expressed her age as Tim's plus twenty years: (T+20). From there we just filled in the chart according to the columns. Always keep expressions with more than one part in parentheses until you're ready to simplify so that you avoid mistakes.

Step 2: Write equations

Our chart does not tell us anyone's age, just the variables of their age. We know that 8 years ago Tim was (T-8) years old and that Kate was (T+12) years old. The problem however tells us that Tim was half as old as Kate 8 years ago. We can set up an equation for this:

(Tim's Age 8 Years ago) = (1/2) x (Kate's age 8 years ago)
T+12= (1/2)(T+12)

Step 3: Solve using algebra

T+12= (1/2)(T+12)
2T – 16 = T + 12
T = 28

Step 4: Evaluate the solution in terms of the problem

Great, we know that T=28, but that is not our answer! If we wrote out ultimate unknown down and circled it, we would know that we are looking for Tim's age 10 years from now. In our chart this is (T+10). Therefore our answer is **38**.

Hidden Integer Constraints

Watch out for problems with hidden real-world constraints. For example, if the problem is asking you how many cars a person has in his garage, the answer cannot be 3.5, because you cannot have just a half of a car.

Sometimes the knowledge of a hidden constraint and a fraction can help you determine an answer. For example, if **23E + 11P = 170**, and E stands for Erasers, and P stands for Pens, then how many erasers and pens are there if there is at least 1 of each?

Because we know that E and P are both integers we can solve this problem. If they didn't have to be integers, there would be no way to solve this equation. To solve, set the equation equal to P and find all the integers that solve this equation resulting in integers for each variable.

P = (170-23E)/11

We see that (170-23E) must be divisible by 11. Make a chart and calculate P using only integers for E. Note, 0 does not count because the prompt tells us there is at least 1 of each.

E	P = Integers?
1	147/11 = NO
2	124/11 = NO
3	101/11 = NO
4	78/11 = NO
5	55/11 = 5 = YES

Note that there are also a finite amount of integers we could have used for E: when E increases, P is decreases.

Hidden Positive Constraints = Possible Algebra

Certain algebra cannot be preformed when we don't know if certain variables are negative or positive. But, when all the quantities in a problem must be positive due to constraints such as those given in the prompt or hidden real world constraints, then we can perform all the algebra we want!

Keep this in mind for DS problems. A common trick is to give the test taker an exponential equation and make him think that the statement is not sufficient because exponential equations lead to two results. However, if one of the results is negative and the other positive, then it could very well be sufficient if the answer has a hidden constraint of having to be positive.

Please check the Equations, Inequalities and VIC Guide for the calculation specifics.

Take Aways

- Try and use the 4 steps of Algebra Translation to lesson errors
- Use charts to organize information
- Never forget to write down and circle what you are really trying to solve for
- Remember the relationship equations
- Try and write variables in terms of other variables to save time solving multiple equations
- Look for hidden constraints, use them to your advantage

Rates & Work: RTD and RTW

These appear often on GMAT test. They come in all different varieties and forms, but the three primary components are always the same:

Rate x Time = Distance or Work

Five common rate problems are (1) basic motion problems, (2) average rate problems, (3) simultaneous motion problems, (4) work problems, and (5) population problems.

The RTD Chart

Always set up a chart for RTD problems. It is a great starting point for any RTD question. Note that the table might not always need a total row or more than one item row.

	Rate	x	Time	=	Distance
Item A					
Item B					
Total					

1. All units in an RTD table MUST match up with each other. If you rate is in *Miles Per Hour* then your time units must be *Hours* and your distance unit must be *Miles*

2. Rates are **always** expressed as DISTANCE/TIME (or WORK/TIME). Always have time on the denominator

Multiple RTD Problems

Difficult rate problems often involve rates, times, and distances for more than one trip or traveler. (1) Set up an **RTD chart** and (2) learn the Rate Relationships so that you can fill in the variables correctly. The thought process is slightly similar to the one in the age example we did in *Algebra Translations* chapter.

Rate Relations

Item A is twice as much, n times as fast as B. (B is half as fast as A)

	Rate	x	Time	=	Distance
Item A	2R				
Item B	R				
Total					

Item A is 1 mile slower than B

	Rate	x	Time	=	Distance
Item A	R – 1				
Item B	R				
Total					

A and B are driving towards each other

	Rate	x	Time	=	Distance
Item A	A				
Item B	B				
	A + B				

Item A is chasing B

	Rate	x	Time	=	Distance
Item A	A				
Item B	B				
Total	Faster – Slower				

Time Relations

A is 30 seconds faster than B (The item with the faster Rate has a lower Time)

	Rate	x	Time	=	Distance
A			T		
B			T+30		

A left the office at the same time as B

	Rate	x	Time	=	Distance
A			T		
B			T		

A and B left at the same time but A arrived home first

	Rate	x	Time	=	Distance
A			T		
B			T + 1		

A left 1 hour after B and arrived home at the same time

	Rate	x	Time	=	Distance
A			T		
B			T + 1		

Sample Situations

The Crash: Leaving together B and A are moving toward each other and will eventually crash into each other

	Rate	x	Time	=	Distance
A	A		T		A's distance
B	B		T		B's distance
	A + B		T		Total D covered

The Quarrel: A and B are moving away from each other

	Rate	x	Time	=	Distance
A	A		T		
B	B		T		
	A – B		T		

The Chase: Car A is chasing car B

	Rate	x	Time	=	Distance
A	A		T		A's distance
B	B		T		B's distance
	Faster – Slower		T		

The Round Trip: A leaves the office then returns the same route

	Rate	x	Time	=	Distance
Going			T going		D
Return			T returning		D
			Total time		2D

The Stalker: A follows B from the office to home (pick numbers if need be)

	Rate	x	Time	=	Distance
A					D
B					D
					2

The Ticket: If A was driving 10 miles per hour slower on the same path

	Rate	x	Time	=	Distance
Actual	R				D
Fake	R – 10				D

Notes

- You can only add the rates for the period during which A and B are both moving
- Whoever is walking for more time should have a larger time expression in the RTD chart

Average Rates: Do not just add and divide!

You need to find the weighted average, not just the simple average.

A has a rate of 5, and B has a rate of 6. Travel same distance. Average rate?

	Rate	x	Time	=	Distance
A	5		FIND		D
B	6		FIND		D
	Solve (WA)		ADD		2D

The RTW Chart (Work Problems)

Work replaces distance in the formula. This refers to the number of jobs completed or the number of items produced. Be aware of any hidden restraints when deciding what your answer means in terms of what is asked.

Figuring Work Rates

You often will have to calculate the rate in work problems (in distance they usually give you it). Here is the rule:

$$\textbf{Work Rate} = \frac{\textbf{Give \# of Jobs}}{\textbf{Given Amount of Time}}$$

or

$$\textbf{Work Rate} = \frac{\textbf{1}}{\textbf{Time to complete 1 job}}$$

Remember (1) that **time is always on the bottom in any work (or distance) problem**, and (2) sometimes the given rate might just be general information like an old rate that can be used to find the new rate.

The Chart Setup

Not all rows may be needed:

	Rate	x	Time	=	Work
A					
B					
Total					

Proportions

You can use proportions for basic work problems as well

- 5 fences/1Hour = X fences/7 Hours

Working Together: Add the rates

If two or more agents are performing simultaneous work, add the rates… Unless one agent undoes the other agent's work. In this case you subtract.

The Population Problem

Although these can be solved by a more advanced method (arithmetic and geometric sequences), they can often be solved by this short cut.

Make a chart showing the time elapsed and the current population by applying the rules for growth as given in the prompt. Here is a sample chart for a population that started at 225 two years ago and doubles every year.

2 years ago	225
1 year ago	450
Now	900
1 year from now	1800
2 years from now	3600

Advanced Population Growth: Exponential Growth (or decay)

You can use the geometric formula, or you can use:

$$\textbf{Population} = S \cdot X^{T/I}$$

I = Interval, amount of time given for quantity to grow
S = Starting value, population at time 0 or "now"
T = Time
X = Amount of growth. If it doubles, then X=2, if it triples X=3, if it half's, then X=1/2

Take Aways

- Set up charts
- Understand the algebra that should be going on in the chart
- Use the examples to help you learn and understand
- Always put time in the denominator of a rate

Ratios

(1) A ratio expresses a particular relationship between two or more quantities.
(2) They can also express a part to part or part to whole relationship. If there are 3 males employees for every 4 females employees, then there is a 3 male employees for 7 employees. That is 3:4 and 3:7 is the part:part and part:whole relationship respectively.
(3) They can both be derived from each other.
(4) You can set directly proportional ratios equal to each other and often solve for a variable that way.
(5) Here are examples of the same ratio written different ways:

- 3:4 = 3/4 = 3 to 4 = 75/100 = .75/1

(6) Keep in mind that the ratios are a relationship, but the actual number of quantity for each item.
(7) Never cancel factors diagonally when you set ratios equal to each other. You only do that in multiplication. You can cancel factors vertically or horizontally.

Unknown Multiplier

Set up a ratio matrix to find unknown multipliers. Like finding the actual number of people that a ratio stands for. *In a library the books are in the ratio of 4 drama to 2 romance to 5 Thrillers. If there are 8 romances, how many total books are there?*

	Drama	Romance	Thriller	Total
Ratio	4	2	5	4+2+5=11
Multiplier				
Real World		8		

Find the hidden multiplier by using the romance row. Now you can do it across every column.

	Drama	Romance	Thriller	Total
Ratio	4	2	5	4+2+5=11
Multiplier	4	2X=8, 8=4	4	4
Real World	4 x 4 = 16	8	5 x 4 = 20	4 x 11 = 44*

*For the real world total you could have also summed the entire real world row (16 + 8 + 20 = 44).

Multiple Ratios: Make a common term

Similar to finding the common denominator of fractions. To combine ratios they need to have a common term.

If the ratio of C to A is 3:2, and the ratio of C to L is 5:4, what is the ratio of C:A:L?

C	:	A	:	L		C	:	A	:	L
3	:	2			Multiply by 5 →	15	:	10	:	
5			:	4	Multiply by 3 →	15	:		:	12
15					This is your answer →	**15**	**:**	**10**	**:**	**12**

The first step was making the shared term C into the common term and finding a common number for it. We did this by using the lease common multiple of 3 and 5 which is 15.

Take Aways

- Ratios are pretty simple
- Use the chart for an unknown multiplier
- Remember the common term method and concept for combining ratios
- Never set two ratios equal to each other and cancel factors vertically like in multiplication

Combinatorics

One of the most complex GMAT topics.

The Fundamental Counting Principle

If you must make a number of separate decisions, then MULTIPLY the numbers of ways to make each *individual* decision to find the number of ways to make *all* the decisions.

The Slot Method

A great way to deal with the counting principle is to c reate a slot for each choice, remembering to fill in those with restrictions first. Each slot is filled with the number of choices possible for that decision. Try to understand this method through the following problem.

You must make a five-digit password for an online bank pin using the digits 0 through 9. The last and first digit of the code must be odd, and you can't use any digit more than once. How many different combinations are possible?

1. We create five slots: _ _ _ _ _
2. Fill in the number of choices for the most restricted first: 5 _ _ _ 4
    a. There are 5 odd numbers (1,3,5,7,9) so we have 5 choices for the first number, 4 for the last
3. Fill in the remaining slots: 5 8 7 6 4
4. Multiply: 5*8*7*6*4 = 6720 choices

Getting Into Combinations and Permutations

The counting principle is really the start of combinations and permutations; the number of ways to arrange items or choose groups. Only those comfortable with this topic should skip the information after this section. Otherwise it is essential for you to learn. The difference between the two is as follows:

Permutations: The number of ways to choose or arrange something if the **order does matter**; choosing (Tim, Dan, and Frank) is not the same as choosing (Dan, Frank, and Tim). In other words, ABC is not the same as BAC. The formula for choosing R items out of N choices is:

$$\frac{N!}{(N - R)!}$$

Combinations: The number of ways to choose or arrange something if the **order does not matter**; choosing (Tim, Dan, and Frank) is the same as choosing (Dan, Frank, and Tim). In other words, ABC is the same as BAC. Thus we factor out those combinations that are the same by adding R! to the bottom of the formula for permutations.

$$\frac{N!}{(N - R)!\ R!}$$

One funny way to remember the difference is thinking about how wrong Master Lock is in calling their products "combination locks" when they are actually "permutation locks".

Simple Factorials

Four people are to sit in 4 chairs lined up in a row. How many different arrangement are there? You could use the simple slot method and fill in 4 3 2 1 however, you can also use *factorials*. Notice that 4*3*2*1, the math you would need to do at the end of the slot method is really *four factorial (4!).*

The number of ways of putting N distinct objects in order, if there are no restrictions, is N!

The Anagram Method Part I: Anagrams

An anagram is a rearrangement of the letters in a word. For instance, the word DISCOUNTED can be arranged to the word DEDUCTIONS or CDDEINOSTU (for our purposes, we include even those anagrams that are gibberish).

For simple words, **ones that don't repeat letters**, you can easily find out the number of anagrams by using simple factorials. The number of anagrams you can make with the 4 letter word GMAT is 4! = 4*3*2*1 = 24 arrangements.

However what if we have a word like PIZZAZZ. If we simply took 7! we would end up with 5040 arrangements. However, this would be over counting as some of the arrangements would be the same because all the Z's are indistinguishable from each other. For example, if you switched the last Z with the first Z, you would still have the same word. The answer is we need to factor out all the ways that we can arrange the Z to give us the same word. Because there are four Z's, we can arrange them in 4! ways, which means we must factor out 4! from 7!.

$$\frac{7!}{4!} = \frac{7*6*5*4*3*2*1}{4*3*2*1} = \frac{7*6*5}{}$$

The word **ATLANTA** follows the same concept. But there are two different letters that repeat this time (in PIZZAZZ there was just 1 letter, Z, that repeated). We can rearrange ATLANTA to AAATTLN to help us look at it.

$$\frac{7!}{3!2!} = \frac{7*6*5*4*3*2*1}{3*2*1*2*1} = \frac{7*6*5*2}{}$$

We must factor out for the 3 A's and the two T's.

- When we have repeated items in the set we reduce the number of arrangements
- **The number of anagrams of a word is the factorial of the total number of letters divided by the factorial(s) corresponding to each set of repeated letters**
- In general: the number of anagrams of a set of items is the factorial of the total number o items divided by the factorial(s) corresponding to sets of indistinguishable items. This one is important for the next lesson…

The Anagram Method Part II: Combinatorics With Repetition & Anagram Grids

You probably wont see an anagram problem on the GMAT. However, the point of doing this work is that most combinatorics problems involving rearranging objects can be solved by anagramming.

1. **Seven people board a bus together with only three seats. How many different seating arrangements are possible if three of the people will sit down?**

The three people who take the seats can be designated as 1, 2, and 3. All of the other four can each be designed as an S for "standing". Therefore, you have the anagram of 123SSSS. 7!/4! Is your answer. You can also set up an anagram grid to help you make the connection (useful for more complicated problems).

Person	A	B	C	D	E	F	G
Seat	1	2	3	S	S	S	S

The top row corresponds to the seven unique people, and the bottom row corresponds to the "seating labels" that were chosen on those people. The anagram word is the bottom part of the grid.

→ You could have also solved this problem using the slot method: The slots-choices for the seats are 7 6 5. Notice also that this is the **permutation** of choosing 3 items from a group of 7.

2. **If the driver of the bus from the previous problem tell all seven passengers who got on together that three of them have won a free ride and don't need to pay, how many different combinations of winners can be selected?**

At first this problem might seem identical to the previous one, however the crucial difference is that the three "chosen ones" are now *also* indistinguishable. Our new anagram is WWWLLLL, where W is for a winning person and L is for a losing person. Our anagram grid looks like:

Person	A	B	C	D	E	F	G
Result	W	W	W	L	L	L	L

And our equation becomes:

$$\frac{\mathbf{7!}}{\mathbf{3! \times 4!}}$$

You also may have noticed that this is the **combination** of choosing 3 items from a group of 7. It is not recommended to use the slot method for this because extra steps would be necessary. Stick to the anagram method.

Multiple Arrangements

If the GMAT requires you to choose two or more sets of items from separate pools, count the arrangements separately – perhaps using a different anagram grid each time. Then multiply the numbers of possibilities for each step.

1. **The GMAT University must select two seniors and two freshmen to represent the school at the National University Olympic ceremony this year. If there are 7 seniors and 6 freshmen at GMAT University, how many different groups of representatives does GMAT University have to choose from?**

This problem has two parts, you must select two members from a group of 12, and 2 members from a group of 11. Our anagrams are as follows.

Senior	1	2	3	4	5	6	7
Result	Y	Y	N	N	N	N	N

Freshmen	1	2	3	4	5	6	
Result	Y	Y	N	N	N	N	

Senior: 7!/2!5!, and freshmen: 6!/2!4!, then multiply the answers together.

Arrangements with Constraints

The most complex combinatorics problems include constraints: one person refuses to sick next to another person, for example.

How many ways can 6 people sit down in 6 chairs in a line if Mike and John, two of the people, refuse to sit next to each other?

1. Find the number of total ways the 6 people can sit without constraints = 6! = 720
2. Find the number of ways the 2 people can sit next to each other
3. Subtract step 2 from step 1

The Glue Method

To find the number of ways 2 people can sit next to each other in any problem, you need to **apply the glue method**, which means you glue them together, whereas they become stuck to each other, and count them as 1 person or unit. But don't forget that John-Mike is not the same as Mike-John if they are glued because in the first one, john is on the left, and in the second one he is on the right.

→ The six people with a glued group of MJ have 5! Ways to sit = 120 ways
→ The six people with a glued group JM have 5! Ways to sit = 120 ways

→ There are a total of 120 + 120, or 240 ways for the John and Mike to sit together.
→ There are 720 – 240 = **480 ways for the 6 people to sit without john or mike together**

Take Aways

- Use the anagram grid whenever you can if you are unsure of what is going on
- Combinations = Less arrangements (order doesn't matter, ABC= BAC, so factor out duplicates)
- Permutations = More arrangements (order matters)

Probability

Probability expresses the likelihood of an event. Probability is **always** a number equal to or between 0 and 1. Zero means the event is not going to happen, and 1 means it will happen 100% of the time.

$$\textbf{Probability} = \frac{\textbf{\# of desired or successful outcomes}}{\textbf{Total number of possible outcomes}}$$

→ The hardest part about probability is realizing that all outcomes must be equally likely for the fraction to be meaningful. You will have to think carefully how to break a situation down into equally likely outcomes.

1. **Flip a coin three times, what's the probability you get heads exactly twice?**

Don't be so fast to say its ½ * ½ * ½ as you would be wrong. It is also tempting to say that you can get 0 heads, 1 head, 2 heads, 3 heads so that is ½ * ½ * ½ * ½ as well, but that is also wrong. All of these outcomes are not equally likely and as we know from above, all of the outcomes need to be equally likely for the probability to make sense.

You have three choices:

1. Write out every possible answer and use that to find the total: HHH, HHT, HTH, THH, HTT, THT, TTH, TTT. Counting, you see 3/8 of the time you will get exactly 2 heads.
2. Construct a counting tree. See Internet for how.
3. Use the binomial "coin flip" distribution formula: In probability, a binomial distribution gives the probabilities of R outcomes in N independent trials for a two-outcome experiment in which the possible outcomes are denoted success and failure. This modified formula only works for a coin flip (or any other event in which the probability of success and failure is 1/2.

$$\mathbf{Pr[R]} = \frac{\mathbf{N!}}{\mathbf{(N-R)!\,R!}} \times (\tfrac{1}{2})^{R}\,(\tfrac{1}{2})^{N-R}$$

R = Number of successes
N = Number of flips

More Than One Event: AND vs OR

1. If you have two independent events X and Y, to determine the probability of event X **AND** event Y will both occur you MULTIPLY the two probabilities together.

What is the probability of a fir coin landing on heads both times when flipped twice? This is a multiplication scenario because the question ask you to find the probability of the first event AND the second event.

2. If you have two mutually exclusive events (meaning the two events cannot both occur at the same time), then the probability of X **OR** Y will occur, you ADD the probabilities together.

What is the probability of rolling either a 4 or a 5 on affair die? This is an addition scenario because you are asked about either the first OR the second. **Had you been asked about the probability of it landing on a 4 the first roll and then a 5 on the second roll, it would be a multiplication scenario.**

A. If an "OR" problem features events that **<u>cannot</u> occur together**, then you can find the OR probability by **adding** the probabilities of the individual events, as we've been doing above.

B. If an "OR" problem features events that **<u>can</u> occur together**, then use the following formula to find the OR probability:

P(A or B) = P(A) + P(B) – P(A and **B)**

The 1 – X Probability Trick

If on a GMAT problem, “success” contains **multiple possibilities** – especially if the wording contains phrases such s **“at least”** and **“at most”** – then consider finding the probability that success **does not happen**. If you can **find this “failure” probability more easily, then you can subtract it from 1**, since (probability of SUCCESS + probability of FAILURE = 1).

What is the probability that on 3 rolls the die will yield a 6? (Using 1 – X strategy)

The probability that it wont yield a 6 is 5/6, so $(5/6)^3$ = 125/216. This is the failure rate. 1 – (125/216) = 91/216, this is our answer.

What is the probability that on 3 rolls the die will yield a 6? (Doing it the regular way)

We would need to find the probability of it happening on the: P(first roll) + P(second roll) + P(Third roll). (1/6) + (5/6)(1/6) + (5/6)(5/6)(1/6) = 91/216

Take Aways

Probability is one of the toughest concepts on the GMAT. The only way to get better at it is to do probability problems, understanding the solution, and mimic that type of work on test day with similar questions. This is the best way to try and understand the thought process. This entire section is important to go through.

Statistics

This is a very simple subject. This guide will keep it short and brief to only the necessities.

Averages

$$\textbf{Average} = \frac{\textbf{Sum}}{\textbf{\# of terms}}$$

Shortcut: if the set is evenly spaced (arithmetic sequence), then you can find the average by simply adding the first and last term together and dividing by two.

Weighted Averages

Not all averages were created equal. Sometimes you have a set in which some data is weighed more than other data. These weights are represented by percents, frequencies, ratios, or fractions. You can view the formula as two ways:

$$\textbf{Weighted Average} = \frac{\textbf{(weight)(data point) + (weight)(data point) +...}}{\textbf{sum of the weights}}$$

or

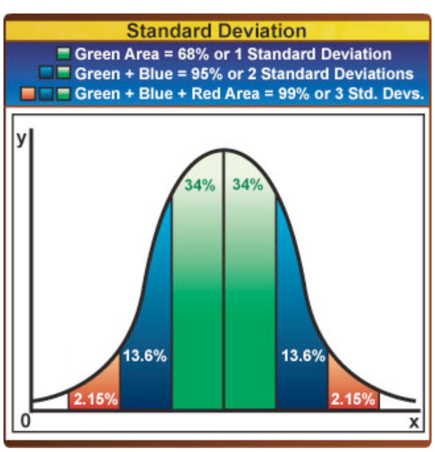
$$\textbf{Weighted Average} = \frac{\textbf{weight}}{\textbf{sum of weights}}\ \textbf{(data point)} + \frac{\textbf{weight}_2}{\textbf{sum of weights}}\ \textbf{(data point}_{2)}\ \textbf{+...}$$

Please also realize that for DS questions, knowing the ratio of the weights can make a statement sufficient most of the time. Simply use the ratio has a fraction.

Standard Deviation

A standard deviation (SD) indicates how far from the average (mean) the data points typically fall. A small SD indicates that a set is clustered closely around the average (mean), and a large SD indicates that the set is spread out widely, with some points appearing far from the mean.

- Remember the numbers 34/14/2… On a normally distribution:
  - 68% of data points will fall within 2 SD of the mean
  - 96% will fall within 4 SD of the mean
  - 98% will fall within 6 SD of the mean



- Always draw a graph like the one on the left. Write, 34, 14, and 2, instead of the ones shown in the graph because you don't need to be exact.

- Variance is just the square of SD. SD = √Variance or, Variance = SD^2

- The only way to mess with the SD is by adding or subtracting data points, **or** changing every data point by a factor. **Simply adding the same number to every data point won't do anything.**

Overlapping Sets