

Chapter 2: Equations

Notes on Simultaneous Equations

-What is the sum of x, y, and x?

$$x+y=8$$

$$x+x=11$$

$$x+y=7$$

you need to think this out. Don't try and substitute etc. Look for shortcuts.

$$x + y = 8$$

$$x + x = 11$$

$$y + z = 7$$

See the pattern? 2 of each variable

$$2x+2x+2z=26$$

Absolute Value

The absolute value refers to the positive value of the expression within the brackets.

Equations that involve absolute values generally have two solutions.

$$|x|=5, \text{ then could be } 5 \text{ or } -5$$

How do you know which is right? You must plug back into original equations to verify

→How to solve for both positive and negative: Page 29 Algebra... unclear, need help.

Chapter 3: Exponents

Base of 0 or 1

$$0^4=0$$

zero raised to any power = 0

$$1^{500}=1$$

1 raised to any power is still just 1

So what if you are told $x=x^2$?

→x must equal 0 or 1

Exponents & Bases

Fractional Base

A regular fraction between 0 and 1 will DECREASE as exponents increase.

→Make some examples... $(3/4)^2=(9/16)$ and then $(3/4)^3=(27/64)$ which is a lot smaller

→Same holds true for decimals between 0 and 1.

Compound Base

$$10^3 = (2 \times 5)^3 = (2)^3 \times (5)^3$$

Also holds true for variables

$$(3x)^4 = (3)^4 (x)^4 = 81x^4$$

Base of -1

→ Any negative base raised to ODD will be NEGATIVE

→ Any negative base raised to EVEN will be POSITIVE

Combining Exponential Terms: Common Bases

The rules in this section apply only when the terms have the same base

all of these rules are related to the fact that exponents are shorthand for repeated multiplication

Multiplying Terms	Add exponents	$z^2 \times z^3 = z^5$
Divide Terms	Subtract Exponents	$(5^6 / 5^2) = 5^4$
Anything raised to 0	is equal to 1	$a^0 = 1 \dots a^3 / a^3 = a^0 = 1$ A BASE RAISED TO ZERO = 1 A BASE RAISED TO ZERO = 1
Negative Exponents	Switch to top or bottom and take out negative (p40 Algebra)	$3^{-3} = 1/3^3$ $1/3^{-3} = 3^3$
Nested Exponents	You are really just doing 1 over same thing with positive exponent Multiply	$(a^5)^2 = a^{5 \times 2} = a^{10}$
Fractional Exponents	Numerator = Power Denominator = Root	

Factoring Out a Common Term

Exponential terms that are added or subtracted **CANNOT** be combined... BUT...

→ If two terms with the **SAME** base are added or subtracted, you can **factor out a common term**

$$11^3 + 11^4 \rightarrow 11^3(1+11) \rightarrow 11^3(12)$$

On the GMAT, it generally pays to factor exponential terms that have common bases

Same Base or Same Exponent

- 1) In problems that involve exponentials on *both* sides of equation, you MUST rewrite the bases so that either the same base or the same exponent appears on both sides of the exponential equation.
- 2) Once you do this, you can usually eliminate the bases or the exponents and rewrite the remainder as an equation

Must be careful if 0, 1, or -1 is the base (or could be the base)... Since these outcomes are not unique. Then this does not work.

Chapter 5: Quadratic Equations

Disguised Quadratics

The GMAT will put disguised quadratics on the test. They love this.

$$3w^2 = 6w$$

Is a favorite disguised quadratic. You cannot just divide by w . You must factor properly.

$$3w^2 = 6w$$

$$3w^2 - 6w = 0$$

$$w(3w - 6) = 0$$

$$w=0 \text{ or } w=2$$

Another example of a favorite disguised quadratic

$$36/b = b-5$$

In general, you can only divide by a variable or an expression if you are 100% certain it is not equal zero.

Zero in the Denominator: Nope

Make sure to look at the original equation to see if one of the quadratics cannot be an answer because it would make the denominator zero.

Chapter 6: Formulas

Strange Symbol Formula

Easy, but just remember if there are challenging ones with the procedure more than once, just perform parenthesis first and work your way outward.

Formulas with Unspecified Amounts

For any formula that does not specific amounts, you want to express the new cost in terms of the original cost.

Step 1: Apply the changes *directly* to the original expression

Step 2: Simplify if necessary while you also...

Step 3: Evaluate

"If cost is tb^4 , then if b is doubled, the new cost is what of the original?"

Step 1: $tb^4 \rightarrow t(2b)^4$

Step 2: $16tb^4$

Step 3: So... that means it is $16 * tb^4$, and tb^4 is the original cost, so that is 1600% of original cost!

Compound Inequalities: Line 'Em Up

Step 1: Simplify equalities and solve to get variable on one side

Step 2: Make all the inequality symbols point the same way...Even if it means making a new row

Step 3: Line up all the common variables in the inequalities [Careful if you have unknown relationships]

Step 3: Check the limiting factor (p93)

Adding Inequalities Together (careful)

-Never subtract

-Never divide

-Can only multiply under certain circumstances

-See page 95, because it is not as straight forward as you think

Common Inequality Trick

The GMAT may ask $mn < 10$, then say statement 1: $M < 2$, and statement 2: $n < 5$... but we don't know, because the symbol could be negative or positive!

Summary: Make sure variables are positive before doing things like this

Inequalities & Absolute Value

$|x + b| = c$ The center is $-b$

"x must be exactly c units away from $-b$ "

$|x + b| < c$ The center is $-b$

"x must be less than c units away from $-b$ "

Use above interpretations and make a graph.

Can solve these algebraically if wanted.

Step 1: First solve regularly

Step 2: Then solve by reversing sign, like a normal |absolute value| problem

$|X - 2| < 5$ what are possible values of X?

Step 1: $|x - 2| < 5$ $= X < 7$

Step 2: $-|x - 2| < 5$ $= -x + 2 < 5$ $= x > 3$

Watch out, common GMAT mistake is to not take negative of entire inside of the |sign|

Chapter 8: Algebra Strategies

Testing Inequality Cases (p119)

B	C	D	Bc<0?	cd>0?
+	-	-	Yes	Yes
-	+	+	Yes	Yes

Special cases to be aware of

STATEMENT	IMPLICATION TO THINK ABOUT
$xy > 0$	X and y are both positive or both negative
$xy < 0$	X and y have different signs (one is positive, one is negative)
$x^2 - x < 0$	$x^2 < x$ [therefore, x is a fraction, or also written: $0 < x < 1$]
See a 0 on one side of eqlty	Consider using positive/negative analysis to help solve the problem