Mastering Important Concepts Tested by GMAT in Triangles-2

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Objective of this Article

In the previous article on triangles, we discussed the following aspects. In case you haven't gone through the previous article, click here .
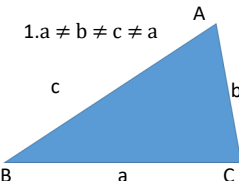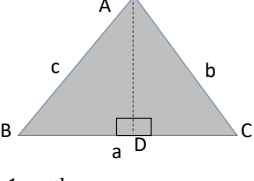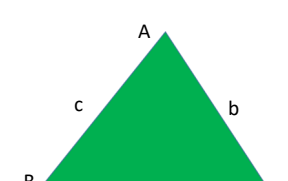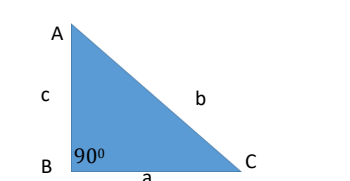
1.   Identifying whether a set of given numbers could be the lengths of the three sides of a triangle,
2.   Range of possible values of the length of one side of a triangle given the lengths of the other two sides, and
3.   Various properties of the <u>angles of the triangle.</u>

With the above knowledge in mind, let us now look at the different types of triangles and how do their properties differ from one another.

Then we will apply the learning on two GMAT like questions.

Properties of different types of triangles:

The table given below lists down the various types of triangles and their properties that are tested on the GMAT. The triangles are sequenced in such a way that the transitions/subtle differences in the corresponding properties are clearly visible.

Properties	Type of Triangles			
	Scalene	Isosceles	Equilateral	Right-Angled
Side Property	•None of the sides are equal. •$a \neq b \neq c \neq$,a where a, b and c are three sides of the triangle.	•Two side are equal. •$a \neq b = c$, where a, b and c are the three sides of the triangle.	•All the sides are equal. •$a = b = c$, where a, b and c are the three sides of the triangle.	•Two sides forming the right angle may or may not be equal. •If they are equal, the triangle is known as isosceles right – angled triangle.
Angle Property	•All the angles are distinct.	•Angles opposite to equal sides are equal.	•All angles are equal to 60^0.	•One of the angles is 90^0.
Area Formula	•Area = ½ x base x height •The above formula is applicable to triangles.		•Area = $\sqrt{3}/4$ x a^2, where 'a' is the length of	•Area = (½) x base x perpendicular
Special Property		•The perpendicular drawn to the unequal side, divides it into two equal parts.	•The perpendicular from any vertex, divides the opposite side into 2 equal halves.	•According to Pythagoras Theorem, $c^2 = a^2 + b^2$, where c is the largest side/hypotenuse
Diagrammatic Representation	1.$a \neq b \neq c \neq a$			

Scalene: 1.$a \neq b \neq c \neq a$

Isosceles: 1. $a \neq b = c$
2.The perpendicular from any vertex, divides the opposite side into 2 equal halves.

Equilateral: 1. $a = b = c$
2. All angles are equal to 60^0.

Right-Angled: 1. $c^2 = a^2 + b^2$, where c is the largest side/hypotenuse
2. Angle ACB = 90^0

Let us walk you through the content of the table one by one and look at the subtle differences that should be kept in mind while solving any question on triangles.

1. Side Property:

- *None of the sides* of a scalene triangle is equal.
- *Any two sides* of an isosceles triangle are equal.
- *All the sides* of an equilateral triangle are equal.

📝 **Note:**

A right-angled triangle can be a scalene triangle or an isosceles triangle. Its depends on whether the lengths of the perpendicular and the base are equal.

Can you tell why can't a right-angled triangle be an equilateral triangle?

2. Angle Property:

- *None of the angles* of a scalene triangle are equal.
- *The two angles opposite the two equal sides* of an isosceles triangle are equal.
- *All the angles* of an equilateral triangle are equal.

Can you notice the relation between the sides and the angles?

- In a scalene triangle where none of the sides are equal, no two angles are equal
- In an isosceles triangle where two sides are equal, the angles opposite to the equal sides are equal.
- In an equilateral triangle where all sides are equal, all angles are equal to 60^0.

📝 **Note:**

A right-angled triangle has one of its angles as 90^0. Since it can be a scalene triangle or an isosceles triangle, the value of the other two angles will depend on what kind of triangle it is.
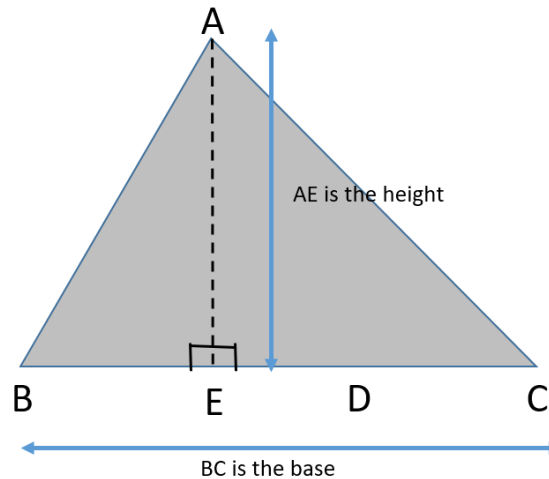
(More will be discussed on this in the third section of the article)

3. Area Property:

The area of any triangle can be found using the formula –

$$\text{Area = } \frac{1}{2} \text{ x base x height}$$

(as shown in the diagram)

AE is the height

BC is the base

For an Equilateral triangle we sometimes use the formula given below to calculate the area quickly: (please note that in an Equilateral triangle all sides and angles are equal)

$$\text{Area = } (\sqrt{3}/4) * a^2$$

Where a is the length of each side of the triangle.

✏️ **Note:**

The area of an equilateral triangle or any special triangle is derived from the basic formula -

$$\text{Area = } \frac{1}{2} \text{ x base x height}$$

Thus, if we remember this formula, it is more than enough to solve any problem related to finding the area of a triangle.

4. Special Property of Triangles:

1 **Isosceles Triangle**

The line joining one vertex of a triangle to the midpoint of the opposite side is called a **median of the triangle.**

In the case of an isosceles triangle, the median and the perpendicular is the same, when drawn from the vertex which joins the two equal sides.

Hence, from the given diagram, we can say that line AD acts as a median, dividing the base BC into two equal halves, and that AD is also perpendicular to BC.

2 Right Angled Triangle

The Right Angled Triangle has three sides known as the perpendicular, base and hypotenuse. The hypotenuse is the largest side and the angle opposite to it is the largest angle and measures 90^0.

We can apply **Pythagoras Theorem**, to define the relation between perpendicular, base and hypotenuse.

From the given diagram, we can write:

$$(\text{Hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$$
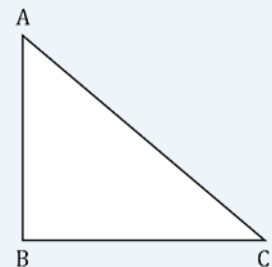
$$\mathbf{AC^2 = AB^2 + BC^2}$$

The summary of these concepts is given in a tabular form at the top of the article. Let us now, with the help of two GMAT like questions understand how to apply the above-mentioned properties.

Illustrative Example 1

In triangle ABC, is $AC^2 = AB^2 + BC^2$?

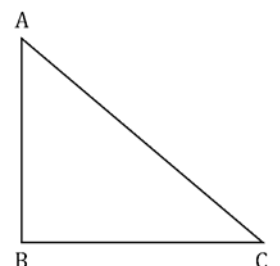1. $\angle BAC + \angle ACB = \angle ABC$
2. $AB = BC$

Explanation:

Steps 1 & 2: Understand Question and Draw Inferences

To find: Is $AC^2 = AB^2 + BC^2$?

Using **Pythagoras theorem**, we know that the above is true <u>if $\angle ABC = 90^0$</u>

<u>Because we can apply the property that we have learned from the table given above that $AC^2 = AB^2 + BC^2$ only when ABC is a right-angled triangle.</u>

Step 3: Analyze Statement 1 independently

Statement 1 tells us that ∠BAC + ∠ACB = ∠ABC

Using Angle Sum Property in triangle ABC, we can write:

$$∠BAC + ∠ACB + ∠ABC = 180^0$$

Substituting the value of ∠BAC + ∠ACB from Statement 1, we have

$$∠ABC + ∠ABC = 180^0$$

$$2(∠ABC) = 180^0$$

Therefore, $∠ABC = 90^0$

Since we could find a unique answer to the question, **Statement 1 alone is sufficient**

Step 4: Analyze Statement 2 independently

Statement 2 tells us that AB = BC

Using the table given above, we can see that this is a property of an isosceles triangle.
Thus, **triangle ABC is an isosceles triangle.**

However, we cannot conclude that ABC is a right-angled triangle because not every isosceles triangle is right-angled.

Hence, this statement is clearly not sufficient to solve the question.

Therefore, **Statement 2 is not sufficient.**

Step 5: Analyze Both Statements Together (if needed)

Since we've already arrived at a unique answer in Step 3, this step is not required.

Answer: **Option A**

Illustrative Example 2

The area of an equilateral triangle is √3 times the area of an isosceles triangle. If the three sides of the isosceles triangle are 5, 5 and 6. Find the length of each side of the equilateral triangle.

A.   4
B.   6
C.   4√3
D.   8
E.   6√3

Solution:

From the triangle property table, we can use two properties to solve this question.

1. Isosceles Special Property: The perpendicular drawn to the unequal side, divides the unequal side into 2 equal halves.

Let's name the isosceles triangle as ABC, where AB = AC = 5 units and BC = 6 units.

The perpendicular from A on BC, will divide BC into 2 equal halves. So BD = DC = 3 units

Using Pythagoras Theorem, we can write $AB^2 = AD^2 + BD^2$
$\implies 5^2 = AD^2 + 3^2$
Therefore, **AD = 4 units.**

Area of triangle ABC = ½ x AD x BC = ½ x 4 x 6 = 12 units.

2. **Area of an equilateral triangle = (√3/4) x side^2**

Since it is given that area of equilateral triangle = √3 x area of isosceles triangle

Therefore, **(√3/4) x side^2 = √3 x 12**
$\implies$ **Side = 4√3**

Hence, the correct answer is Option C.

We hope these questions helped you in understanding the importance of the properties of triangles and their application in GMAT context.

Try to remember these properties in a methodical manner as given in the table and we are sure that you will be able to apply them successfully in all questions related to triangles.

NOTE: This is the third article in our Geometry series. If you liked the above article, you can also go through the link given below. I am sure you will find it useful

Common Mistakes in Geometry

Mastering Important Concepts Tested by GMAT in Triangles – I

Mastering Important Concepts Tested by GMAT in Triangle – III

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