Math Facts Summary

Multiplication Table

I know this may seem insulting, but a trip back to third grade is almost always necessary. Excel has sapped you mathematical strength, and smart phones make calculation anywhere and anytime painless. We've bolded and highlighted the facts that statistically give third graders and adults the most trouble.

×	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	**24**	27	30	33	36
4	4	8	12	16	20	24	**28**	**32**	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	**60**
6	6	12	18	24	30	**36**	**42**	**48**	**54**	60	66	72
7	7	14	21	**28**	35	**42**	**49**	**56**	63	70	77	**84**
8	8	16	**24**	**32**	40	**48**	**56**	**64**	72	80	88	**96**
9	9	18	27	36	45	**54**	63	72	81	90	99	**108**
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	**132**
12	12	24	36	48	**60**	72	**84**	**96**	**108**	120	**132**	144

By the way, there are plenty of smart-phone apps for basic math facts. Most of them are free, so if you have a religious objection to traditional flash cards, you might want to download such an app. Unfortunately, there isn't an app for all of the other facts you have to memorize…yet.

Operations with Evens and Odds:

Sums (and differences) and products of evens and odds follow some very simple rules.

Add/Subtract	EVEN	ODD
EVEN	**EVEN**	**ODD**
ODD	**ODD**	**EVEN**

Multiply	EVEN	ODD
EVEN	**EVEN**	**EVEN**
ODD	**EVEN**	**ODD**

Some Comments:

- Only integers can be even or odd.
- Zero is an even number.
- Two is the only even prime.
- When in doubt check one example to confirm the rule.
- One even factor in a product makes the product even. There is no similar table for division.

Multiplication and Division of Positive and Negative Integers:

The rules for multiplication and division are the same.

Multiply/Divide	**POSITIVE**	**NEGATIVE**
POSITIVE	POSITIVE	NEGATIVE
NEGATIVE	NEGATIVE	POSITIVE

Multiplication of integers with the same sign will result in a positive and the multiplication of integers with different signs will result in a negative.

Powers of Primes

These come in very handy in a number of contexts – so many in fact that it would be pointless to try to list them all. You need to know these backward and forwards. We mean that literally. It's not enough to know that $2^7 = 128$. When you see 128, you need to know it's 2^7. Powers of 2 are especially important because almost all geometric growth problems are doubling problems – grasshoppers/bacteria/elves double their population every x minutes/hours/eons, etc.

Powers of 2	Powers of 3	Powers of 5	Powers of 7
$2^1 = 2$	$3^1 = 3$	$5^1 = 5$	$7^1 = 7$
$2^2 = 4$	$3^2 = 9$	$5^2 = 25$	$7^2 = 49$
$2^3 = 8$	$3^3 = 27$	$5^3 = 125$	$7^3 = 343$
$2^4 = 16$	$3^4 = 81$	$5^4 = 625$	
$2^5 = 32$	$3^5 = 243$	$5^5 = 3125$	
$2^6 = 64$			
$2^7 = 128$			
$2^8 = 256$			
$2^9 = 512$			
$2^{10} = 1024$			

Squares

It pays to know the squares of positive integers up to 25. Again, backward and forwards.

$1^2 = 1$	$6^2 = 36$	$11^2 = 121$	$16^2 = 256$	$21^2 = 441$
$2^2 = 4$	$7^2 = 49$	$12^2 = 144$	$17^2 = 289$	$22^2 = 484$
$3^2 = 9$	$8^2 = 64$	$13^2 = 169$	$18^2 = 324$	$23^2 = 529$
$4^2 = 16$	$9^2 = 81$	$14^2 = 196$	$19^2 = 361$	$24^2 = 576$
$5^2 = 25$	$10^2 = 100$	$15^2 = 225$	$20^2 = 400$	$25^2 = 625$

Square Roots

If you know your squares, you know these too, but just for good measure…

$\sqrt{1} = 1$	$\sqrt{36} = 6$	$\sqrt{121} = 11$	$\sqrt{256} = 16$	$\sqrt{441} = 21$
$\sqrt{4} = 2$	$\sqrt{49} = 7$	$\sqrt{144} = 12$	$\sqrt{289} = 17$	$\sqrt{484} = 22$
$\sqrt{9} = 3$	$\sqrt{64} = 8$	$\sqrt{169} = 13$	$\sqrt{324} = 18$	$\sqrt{529} = 23$
$\sqrt{16} = 4$	$\sqrt{81} = 9$	$\sqrt{196} = 14$	$\sqrt{361} = 19$	$\sqrt{576} = 24$
$\sqrt{25} = 5$	$\sqrt{100} = 10$	$\sqrt{225} = 15$	$\sqrt{400} = 20$	$\sqrt{625} = 25$

On the GMAT all square roots are positive unless they are used to solve an equation. The square root of 4 is 2, but if $x^2 = 4$, then $x = 2$ or -2

Some Ugly Square Roots

Every integer has a square root, but most don't have a clean square root. For example:

$$\sqrt{13} = 3.605551275463989 \ldots$$

The decimal expansion goes on forever and never repeats. The technical name for this is "irrational." It turns out that the square root of any integer that isn't a perfect square is irrational. Sounds bad, right? Not to worry – you only have to know three approximations:

$$\sqrt{2} \approx 1.4, \sqrt{3} \approx 1.7, \sqrt{5} \approx 2.25$$

Cubes

Some cubes are helpful too.

$1^3 = 1$	$2^3 = 8$	$3^3 = 27$	$4^3 = 64$	$5^3 = 125$	$6^3 = 216$

Cube Roots

$\sqrt[3]{1} = 1$ $\sqrt[3]{8} = 2$ $\sqrt[3]{27} = 3$ $\sqrt[3]{64} = 4$ $\sqrt[3]{125} = 5$ $\sqrt[3]{216} = 6$

$\sqrt[3]{-1} = -1$ $\sqrt[3]{-8} = -2$ $\sqrt[3]{-27} = -3$ $\sqrt[3]{-64} = -4$ $\sqrt[3]{-125} = -5$ $\sqrt[3]{-216} = -6$

Divisibility Rules

The rules can be broken down into 5 groups:

- 2, 4, and 8: Powers of 2
- 3 and 9: Multiples of 3
- 5 and 10: Multiples of 5
- 6: The mother of all divisibility rules

However we can use the rules below to construct rules for lots of composite numbers. I'll point out a couple of examples as we go along.

Divisibility Rules for 2, 4, and 8

1) A number is divisible by 2 if
   a) **The last digit is divisible by 2.** For example 2,106 is divisible by two because the last digit, 6, is divisible by 2, but 2,119 is not divisible by 2 because 9 is not divisible by two.
2) A number is divisible by 4 if
   a) **The last two digits form a number that is divisible by 4.** For example 3,436 is divisible by 4 because 36 is divisible by 4, but 3,774 is not divisible by 4 because 74 is not divisibly by 4. Note that if the digit in the ten's place is a zero the rule still applies; 104 and 2,308 are both divisible by 4.
   b) **It is divisible by 2 twice.** For example, $256 \div 2 = 128$, and $128 \div 2 = 64$, so 256 is divisible by 4, but 170 is not divisible by 4 because $170 \div 2 = 85$, and 85 is not divisible by 2. Thus, 170 is only divisible by 2 once, so it is not divisible by 4.
3) A number is divisible by 8 if
   a) **The last three digits form a number that is divisible by 8.** For example 45,856 is divisible by 8 because 856 is divisible by 8, but 45,764 is not divisible by 8 because 764 is not divisible by 8. Again, if zeros appear in the hundreds or tens place, the rule still applies; 45,056 and 45,008 are both divisible by 8.
   b) **The number is divisible by 2 three times.** For example 128 is divisible by 8 because $128 \div 2 = 64$, $64 \div 2 = 32$, and $32 \div 2 = 16$. This rule is not as practical as the similar rule for 4, but it will work in a pinch.

There are similar rules for all powers of two – for 2^n, check the the number formed by the last n digits for divisibility by 2^n.

Divisibility Rules for 3 and 9

1) **A number is divisible by 3 if the sum of the digits is divisible by 3.** For example, 432 is divisible by 3 because $4 + 3 + 2 = 9$ and 9 is divisible by 3, but 76,517 is not divisible by 3 because $7 + 6 + 5 + 1 + 7 = 26$, and 26 is not divisible by 3.
2) **A number is divisible by 9 if the sum of the digits is divisible by 9**. For example, 6,588 is divisible by 9 because $6 + 5 + 8 + 8 = 27$ and 27 is divisible by 9, but 751 is not divisible by 9 because $7 + 5 + 1 = 13$, and 13 is not divisible by 9.

Divisibility Rules for 5 and 10

1) **A number is divisible by 5 if it ends in a 5 or a zero**. For, example 5, 25, 125, and 625 are all divisible by 5 because they end in five, and 10, 100, and 650 are divisible by 5 because they end in zero.
2) **A number is divisible by 10 if it ends in zero.** For example 550, 760, 10,000, and 5,000,000 are all divisible by 10 because they end in zero.

There's a rule for 15 that we'll discuss in the next chapter. You should try to figure it out for yourself – think about the progression of the 10's digit.

Divisibility Rule for 6

A number is divisible by 6 if it divisible by 2 and 3. For example, 312 is divisible by 6 because it is divisible by 2 and 3 (if you don't know why 312 is divisible by 2 and 3, see above and apply a ruler to your knuckles).

Fraction Decimal Equivalents

Time out for vocabulary:

- Numerator = the integer above the fraction bar.
- Denominator = the integer below the fraction bar.

All fractions have decimal equivalents that either terminate (come out nice and clean when you divide the numerator by the denominator) or repeat (keep giving you the same number or pattern of numbers when you divide the numerator by the denominator). Numbers or patterns of numbers that repeat have a bar over the repeating pattern.

$\frac{1}{2} = 0.5$	$\frac{1}{3} = 0.\overline{3}$	$\frac{1}{4} = 0.25$	$\frac{1}{5} = 0.2$	$\frac{1}{8} = 0.125$	$\frac{1}{20} = 0.05$	$\frac{1}{10} = 0.1$
	$\frac{2}{3} = 0.\overline{6}$	$\frac{3}{4} = 0.75$	$\frac{2}{5} = 0.4$	$\frac{3}{8} = 0.375$	$\frac{1}{25} = 0.04$	$\frac{1}{100} = 0.01$
			$\frac{3}{5} = 0.6$	$\frac{5}{8} = 0.625$	$\frac{1}{50} = 0.02$	
			$\frac{4}{5} = 0.8$	$\frac{7}{8} = 0.875$		

The table is organized by denominator. A few denominators are missing – 6, 7, 9. Six isn't that hard, but a little unnecessary. Seven is beyond unnecessary. Nine has an interesting pattern, as does 11, but the payoff for these is negligible. Feel free to memorize more equivalents, but you absolutely have to know the ones above.