

## Lesson 57

**AIM:** How can we use permutations to solve probability problems?

**PERFORMANCE STANDARDS:** M4, M5, M7

**PERFORMANCE OBJECTIVES:** The students will be able to...

1. Explain how permutations can be used to count the number of outcomes of an event and the number of elements in a sample space.
2. Apply the permutation formulas to solve numeral and word probability problems.

**VOCABULARY:** No new terms

**MOTIVATION:**

A variable,  $x$  or  $y$ , is randomly selected in order from each parentheses:  $(x + y)$   $(x + y)$   $(x + y)$  and written as a product. For instance,  $xyx$  and  $xyy$  are two possible products. How many such orders are possible? Recall the fundamental counting principle to obtain  $2 \times 2 \times 2 = 2^3$ . Write all such orders.

What is the probability that the product is  $x^3$ ? ( $\frac{1}{8}$ )

What is the probability that the product is  $x^2y$ ? ( $\frac{3}{8}$ )

**DEVELOPMENT:**

1. The last question should lead to consideration of terms like  $xyx$ ,  $xyy$ ,  $yxx$ . Note that these are permutations which contain repetitions. **Ask:** “What

formula would count these for us?” ( $\frac{n!}{r!} = \frac{3!}{2!} = 3$ )

Therefore the probability that the product is equal to  $x^2y$  is  $\frac{3}{8}$ . In this example the number of elements in the sample space were given to us by formula for permutations. Further examples involving the use of permutations in probability problems may be included such as the following: The letters  $aabcd$  are arranged at random. **Ask:** “What is the probability that the two  $a$ ’s will be in the last two positions?”

Note here the number of permutations is  $\frac{5!}{2!} = 60$ . The number of “words” ending with  $aa$  are  $3 \times 2 \times 1 = 6$ .

Therefore the answer is  $\frac{6}{60} = \frac{1}{10}$ .

**APPLICATIONS:**

1. A three-digit numeral is formed by selecting from the digits 2, 3, 4, 7 with no repetitions. **Ask:** “What is the probability that the number formed is greater than 300?”

Students should see that the answer,  $\frac{3}{4}$ , can be obtained immediately by using the digits 3, 4, 7 as the hundreds digit. The answer can also be obtained by considering the total number of such 3-digit numbers as  $4 \times 3 \times 2 = 24$ . The numbers greater than 300 must begin with either 3, 4 or 7 followed by any one of three other digits, etc.  $= 3 \times 3 \times 2 = 18$ . Therefore,  $\frac{18}{24}$  is the required answer.

2. Two cards are drawn from a standard deck of playing cards without replacement.

a) Express the number of elements in the sample space using the notation

$${}_nP_r \cdot ({}_52P_2)$$

b) What is the probability that the two cards selected are aces?

$$\frac{{}_4P_2}{{}_52P_2} = \frac{1}{221}$$

c) What is the probability that the two cards are the Ace and King of hearts in that order?

$$\frac{{}_1P_1 \cdot {}_1P_1}{{}_52P_2} = \frac{1}{52 \cdot 51}$$

3. An urn contains 7 blue marbles, 3 yellow marbles, and 5 green marbles. Three marbles are selected from the urn without replacement. Find the probability that the marbles selected are 2 blue marbles and 1 green marbles in that order.

$$\frac{({}_7P_2 \cdot {}_5P_1)}{{}_{15}P_3}$$

4. If there are two brothers in a group of 5 boys and these boys are lined up at random, what is the probability that the brothers will be next to each other?

$$\frac{4!2!}{{}_5P_5}$$

### UNIT PROJECT:

Design and solve a permutation task related to your game of chance.

Black Jack: Two cards are drawn from a standard deck of cards without replacement. Find the probability that the cards are one picture card and one ace in that order.

### MATH A-TYPE PROBLEM:

Two cards are drawn without replacement from a standard deck of cards. Find the probability that both cards are diamonds.

## SUMMARY:

1. How can the probability of an event be computed? (We must take the ratio of the number of ways an event may occur to the number of total possible outcomes. To find these, we need the fundamental counting principle and the formulas for permutations.)
2. What Mathematics Standards did we address today?

## HOMEWORK:

1. What is the total number of ways a coin that is tossed 5 times may fall? (25)
2. What is the probability of tossing a coin 5 times and obtaining HTHTT in exactly that order? ( $\frac{1}{25}$ )
3.
  - a) In how many orders may a coin show 3 tails and 2 heads after being tossed 5 times? ( $\frac{5!}{3!2!}$ )
  - b) Suppose we know there are three tails and two head, what is the probability that the order was exactly HTHTT?  $1 \div (\frac{5!}{3!2!})$
4. Three cards are drawn from a standard deck of playing cards without replacement. Find the probability that the cards are one picture card and two aces in that order.  
$$\frac{{}_{16}\underline{P}_1 \cdot {}_4\underline{P}_2}{{}_{52}\underline{P}_3}$$
5. The letters of the alphabet are rearranged randomly.  
What is the probability the first five letters are vowels? ( $\frac{5! \cdot 21!}{26!}$ )